Thermal unpairing transitions affected by neutrality constraints and chiral dynamics

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Abstract. We discuss the phase structure of homogeneous quark matter under the charge neutrality constraints, and present a unified picture of the thermal unpairing phase transitions for a wide range of the quark density. We supplement our discussions by developing the Ginzburg-Landau analysis.

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Surprisingly rich phases of quark matter under high pressure are being revealed by extensive efforts [1, 2, 3, 4, 5, 6]. The key ingredients which are important at moderate density can be divided into two categories; (i) the dynamical effects and (ii) the kinematical effects. The dynamical effects include strong coupling effects [7, 8, 9, 10], an interplay between the pairing and chiral dynamics [3, 4, 5, 6] and so on. In this talk, we focus on the competition of the pairing with the chiral dynamics and the kinematical effects such as (1) the strange quark mass and (2) the charge neutrality constraints under the β-equilibrium. The kinematical effects will make a stress on the pairing and bring about the exotic phase called "*gapless*" superconductivity at moderate density [1, 2]; the gapless CFL (gCFL) [2] is one of such examples. The neutrality constraints are also known to lead to an interesting complication in the phase diagram even at finite temperature: For instance, the Ginzburg-Landau analysis [11] shows that the down-quark pairing phase (dSC phase) consisting of only the *u*-*d* and *d*-*s* pairings may become the second coldest phase at high density. However, this conclusion seems to be model-dependent. In fact, the NJL analyses which incorporate the chiral dynamics [5, 6] show that the second densest phase is the up-quark pairing (uSC) phase consisting of only the *u*-*d* and *u*-*s* pairing. In this talk, we report our recent work [4] which gives a systematic and unified picture of the thermal unpairing transitions under the charge neutrality constraints for a wide range of the quark density.

We start with the Lagrangian density $\mathscr{L} = \bar{q}(i\partial\!\!\!/ - \hat{m}_0 + \hat{\mu}\gamma_0)q + \mathscr{L}_{\mathrm{int}}$ with $\mathscr{L}_{\mathrm{int}}$ being the following 4-fermion coupling [3]

$$\mathscr{L}_{\mathrm{int}} = G_D \textstyle\sum_{\eta=1}^{3} \left[(\bar{q}P_\eta^t \bar{q})({}^t q\bar{P}_\eta q)\right] + G_S \textstyle\sum_{\alpha=0}^{8} \left[(\bar{q}\lambda_{F\alpha} q)^2 + (\bar{q}i\gamma_5\lambda_{F\alpha} q)^2\right]. \qquad (1)$$

The first term simulates the attractive interaction in the color (flavor) anti-triplet and $J^P = 0^+$ channel in QCD, i.e., $(P_\eta)_{ij}^{ab} = i\gamma_5 C\varepsilon^{\eta ab}\varepsilon_{\eta ij}$. See [4], for the other details of notation. We take the chiral SU(2) limit $\hat{m}_0 = \mathrm{diag.}\{0,0,m_s\}$ with $m_s = 80\,\mathrm{MeV}$ fixed. $\hat{\mu}$ in the Lagrangian contains the charge chemical potentials (μ_e, μ_3, μ_8) which couple to the electric and two diagonal color charge densities as

$$\hat{\mu}_{ij}^{ab} = \mu - \mu_e Q_{ij} + \mu_3 T_3^{ab} + \mu_8 T_8^{ab} + (\text{off-diagonal part}), \qquad (2)$$

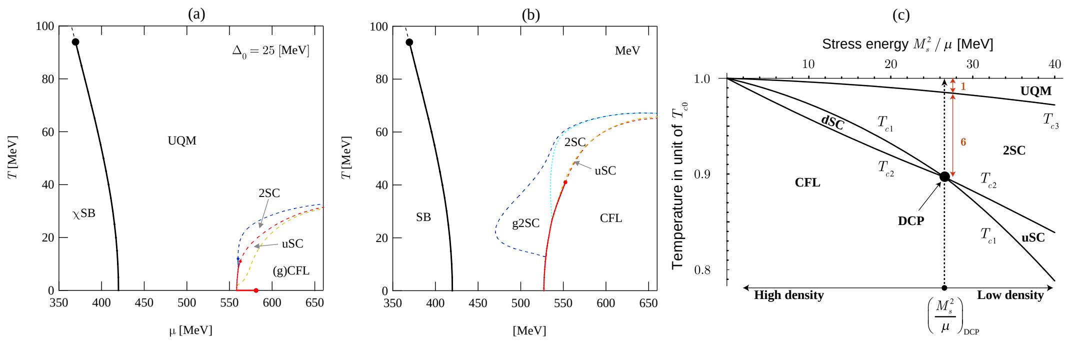

FIGURE 1. Phase diagram calculated with the extremely weak (qq) coupling (a) and that with weak coupling (b). In (a), there is a small region for the realization of gCFL phase at $T = 0$ (bold red line). (c) shows $T_{c\eta}$ evaluated by the Ginzburg-Landau analysis with the parameter set $G_D/G_S \cong 0.42$ and $\mu = 500\,\text{MeV}$. When the coupling strength is increased, $(M_s^2/\mu)_{\rm DCP}$, the intersection of T_{c2} and T_{c1} moves to higher value, while the ratio of $T_{c0} - T_{c3} : T_{c0} - T_{\rm DCP} = 1 : 7$ would not be affected.

where we have defined Q, T_3 and T_8 as usual [4]. It can be shown that the chemical potentials for off-diagonal color densities are unnecessary for the standard diagonal ansatz for the diquark condensate, i.e., $\langle q_i^a q_j^b \rangle \sim (P_\eta)_{ij}^{ab}$ with $\eta = 1,2,3$ [4]. We determine the phases in (μ, T)-plane by calculating the effective potential through the mean field approximation with the condensate fields, $\Delta_\eta = \frac{G_d}{8}\langle {}^t q P_\eta q \rangle$, and $\hat{M} - \hat{m}_0 = -\frac{G_s}{2N_c}\text{diag.}(\langle \bar{u}u \rangle, \langle \bar{d}d \rangle, \langle \bar{s}s \rangle)$. We perform the calculation with several values of G_D/G_S with G_S set so as to reproduce the dynamical quark mass $M = 400\,\text{MeV}$ in the chiral limit at $\mu = T = 0$ for a cutoff $\Lambda = 800\,\text{MeV}$; we have $G_S\Lambda^2 = 2.17$. Fig. 1(a) and (b) show the phase diagram for $G_D/G_S \cong 0.42$ (*the extremely weak coupling case*) and that for $G_D/G_S \cong 0.63$ (*the weak coupling case*). The χSB denotes the chiral-symmetry broken phase and the UQM is an abbreviation of the "*unpaired quark matter*". For other phases, see [4]. Each of the superconducting phases, CFL, 2SC and uSC, has its gapless version, gCFL, g2SC, and guSC, where some quasi-quarks become gapless in the presence of the finite background charge chemical potentials. As the value of the diquark coupling is increased, these "*premature*" gapless phases tend to disappear, and the fully gapped phases dominate the phase diagram. In fact, we can see that the gCFL phase at $T = 0$ in Fig. 1(a) is taken over by the UQM phase in (b), which can be interpreted as a consequence of the competition between the dynamical and kinematical effects [4].

The reason why the uSC shows up in the phase diagram instead of the dSC can be nicely understood by the Ginzburg-Landau (GL) analysis. We can expand the effective potential in terms of the gap parameters near the critical temperature T_{c0} which denotes the CFL $\longrightarrow$ UQM transition temperature in the symmetric matter with $m_s = 0$:

$$\mathscr{L}_{\rm GL} = 4N[\mu]\big\{ -f_i(M_s)\Delta_i^2 + \tfrac{1}{2}g_{ij}(M_s)\Delta_i^2\Delta_j^2 + \cdots \big\}, \tag{3}$$

where $N[\mu] = \mu^2/2\pi^2$ is the density of state. It is important that under the neutrality constraints, the GL coefficients become functions of M_s, which may be expanded as

$$f_i(M_s, T) = a_{0i} + a_{2i}M_s^2 + a_{4i}M_s^4 + \cdots, \quad g_{ij}(M_s, T) = \beta_{0ij} + \beta_{2ij}M_s^2 + \cdots. \tag{4}$$

We can calculate all the coefficients using the Feynman diagrams. The ultraviolet divergence appears only in a_{0i} as a consequence of the singularity in the diquark propagator at $T = T_{c0}$ for $M_s = 0$. We can regularize it by subtracting the equation expressing the Thouless condition which serves as the mass counter term that guarantees the second order transition at $T = T_{c0}$. We can show $a_{0i} = -t$ $(i = 1,2,3)$ with t being the reduced temperature $\frac{T-T_{c0}}{T_{c0}}$, and $\beta_{0ij} = \delta_{ij}\frac{7\zeta(3)}{16\pi^2 T_{c0}^2}$. a_{2i} was derived in [11], and a_{4i}, (β_{2ij}) was obtained[1] in [4]. As we show below, a_{2i} (a_{4i}, β_{2ij}) causes a split of the order of M_s^2 (M_s^4) in the melting temperature; $T_{c0} \to (T_{c1}, T_{c2}, T_{c3})$ where Δ_η vanishes at $T_{c\eta}$. When T is increased, the first CFL-to-*non*-CFL transition with $\Delta_{\eta_1} \to 0$ takes place at $T = T_{c\eta_1}$. We can determine $T_{c\eta_1}$ by solving $T_{c\eta_1} = \min.\{T_1, T_2, T_3\}$ with T_η defined by the root of $\Delta_\eta^2(T_\eta) = 0$ where $\Delta_\eta^2(T) = g_{\eta j}^{-1} f_j(M_s, T)$ is the solution of $\left.\frac{\partial \mathscr{L}_{\rm GL}}{\partial \vec{\Delta}}\right|_T = \vec{0}$. We have

$$\frac{T_\eta}{T_{c0}} \;=\; 1 + a_{2\eta} M_s^2 + \Big(a_{4\eta} + \frac{7\zeta(3)}{16\pi^2 T_{c0}^2} \textstyle\sum_j \beta_{2\eta j}^{-1}(a_{2\eta} - a_{2j})\Big) M_s^4 + \cdots, \tag{5}$$

by which $T_{c\eta_1}$ as a function of M_s can be determined. To find the next melting order parameter Δ_{η_2}, we put $\Delta_{\eta_1} = 0$ into Eq. (3) and repeat the same procedure in two order parameter space. Finally, we obtain the order of hierarchical melting transitions $(T_{c\eta_1} < T_{c\eta_2} < T_{c\eta_3})$. We confirmed that Δ_3 survives at highest temperature so that $\eta_3 = 3$ irrespective of the value of M_s. In contrast, which of Δ_1 and Δ_2 first vanishes with increasing T depends on M_s; in fact, we have found the *doubly critical* strange quark mass $M_s^{\rm DCP}$ above (below) which the uSC (dSC) is realized as the second coldest phase. In Fig. 1(c), we have given the phase diagram in the $(M_s^2/\mu, T)$-plane, obtained by the GL analysis for $G_D/G_S \cong 0.42$ and $\mu = 500\,\mathrm{MeV}$.

In conclusion, we have made an extensive analysis of the phase diagram of the quark matter and given a unified view on the thermal unpairing transitions. By extending the earlier work [11] with the higher order effects of the strange quark mass on the pairing taken into account, we have shown how the window for the dSC-realizaion in the high density regime tends to close towards lower density. It should also be stressed that an analytic expression for the doubly critical point [12] can be derived in our framework.

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[1] One has to take care that there is a feedback to β_{2ij} from the Fermi gas part of thermodynamic potential.